Inference rules cheatsheet

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1 Regole per i connettivi proposizionali

Queste regole descrivono il comportamento di tutti i connettivi proposizionali:

$$\vdash, \wedge, \vee, \neg, \bot, \rightarrow$$

Axiom

$$\phi \vdash \phi \tag{1}$$

Weakening

$$\frac{\Gamma \vdash \phi}{\Gamma, \alpha \vdash \phi} \tag{2}$$

$(\vdash \wedge)$

$$\frac{\Gamma \vdash \alpha \quad \Gamma \vdash \beta}{\Gamma \vdash \alpha \wedge \beta} \tag{3}$$

$(\wedge \vdash)$

$$\frac{\Gamma, \alpha \vdash \gamma}{\Gamma, \alpha \wedge \beta \vdash \gamma} \qquad \frac{\Gamma, \beta \vdash \gamma}{\Gamma, \alpha \wedge \beta \vdash \gamma} \tag{4}$$

$(\vdash \vee)$

$$\frac{\Gamma \vdash \alpha}{\Gamma \vdash \alpha \vee \beta} \qquad \frac{\Gamma \vdash \beta}{\Gamma \vdash \alpha \vee \beta} \tag{5}$$

$(\vee \vdash)$

$$\frac{\Gamma, \alpha \vdash \gamma \quad \Gamma, \beta \vdash \gamma}{\Gamma, \alpha \vee \beta \vdash \gamma} \tag{6}$$

$(\vdash \rightarrow)$

$$\frac{\Gamma, \alpha \vdash \beta}{\Gamma \vdash (\alpha \rightarrow \beta)} \tag{7}$$

Modus Ponens

$$\frac{\Gamma \vdash \alpha \quad \Gamma \vdash (\alpha \rightarrow \beta)}{\Gamma \vdash \beta} \tag{8}$$

$(\vdash \bot)$

$$\frac{\Gamma \vdash \alpha \quad \Gamma \vdash \neg\alpha}{\Gamma \vdash \bot} \tag{9}$$

(⊥)

$$\frac{\Gamma \vdash \bot}{\Gamma \vdash \alpha} \tag{10}$$

(¬ ⊢)

$$\frac{\Gamma \vdash \alpha}{\Gamma, \neg\alpha \vdash \bot} \tag{11}$$

(⊢ ¬)

$$\frac{\Gamma, \alpha \vdash \bot}{\Gamma \vdash \neg\alpha} \tag{12}$$

Reductio ad Absurdum

$$\frac{\Gamma, \neg\alpha \vdash \bot}{\Gamma \vdash \alpha} \tag{13}$$

2 Regole per i connettivi predicativi

Queste regole di inferenza definiscono il comportamento dei quantificatori nelle formule predicative. Sono riportate sia nella notazione con ipotesi implicite, sia nella notazione con ipotesi esplicite.

Esistenza

$$\frac{\begin{array}{c}\vdots\\P(a)\end{array}}{\exists x P(x)} \qquad \frac{\Gamma \vdash P(a)}{\Gamma \vdash \exists x P(x)} \tag{14}$$

Scelta

$$\frac{\begin{array}{c}\vdots\\\forall x P(x)\end{array}}{P(a)} \qquad \frac{\Gamma \vdash \forall x P(x)}{\Gamma \vdash P(a)} \tag{15}$$

Generalizzazione Dato $a \notin \Gamma, P(x)$

$$\frac{\begin{array}{c}\vdots\\P(a)\end{array}}{\forall x P(x)} \qquad \frac{\Gamma \vdash P(a)}{\Gamma \vdash \forall x P(x)} \tag{16}$$

Estensione Dato $a \notin \Gamma, \phi$

$$\frac{\begin{array}{c}P(a)\\\vdots\\\phi\end{array}}{(\exists x P(x)) \to \phi} \qquad \frac{\Gamma, P(a) \vdash \phi}{\Gamma, \exists x P(x) \vdash \phi} \tag{17}$$