## EXERCISES OF WEEK FOUR (2014/09/29, 11:00AM)

Exercise 1. Given three vectors $a, b, c \in E_{3}$, let $A$ be the matrix defined column-wise

$$
A:=(a|b| c) .
$$

Show that $\operatorname{det}(A)=a \cdot(b \times c)$.

## Exercise 2. Let

$$
\ell_{1}:=\ell(P, v), \quad \ell_{2}:=\ell(Q, w)
$$

be two non-degenerate lines such that $v \times w=0$. Show that either

$$
\ell_{1}=\ell_{2} \text { or } \ell_{1} \cap \ell_{2}=\varnothing .
$$

Exercise 3. Suppose that we have two non-degenerate lines

$$
\ell:=\ell(P, v), \quad \ell^{\prime}:=\ell(Q, w) .
$$

in the plane. We can define a distance between $\ell$ and $\ell^{\prime}$

$$
d\left(\ell, \ell^{\prime}\right):=\inf \left\{d\left(R, R^{\prime}\right) \mid R \in \ell, R^{\prime} \in \ell^{\prime}\right\}
$$

Try to express the distance in terms of $P, Q, v, w$.
Exercise 4. Find the area of the polygon with vertices given by the points

$$
P(0,0), \quad Q(2,3), \quad R(5,6), \quad T(1,5) .
$$

