San Francisco State University
Faculty of Science
Department of Physics and Astronomy

A particular study about Big Bang Nucleosynthesis: today's Deuterium rate.

Professor:
Andisheh Mahdavi

Federico Bobbio

Spring 2013

Abstract

In this paper we are going to analyse today's Deuterium rate depending on today baryion density. Using data collected by Wagoner et alia concerning the reactions of isotopes of Hydrogen and Helium during nucleosynthesis we are going to build the differential equation system that will enable us to reach our goal. The program used to analyse the model is *Wolfram Mathematica 6*, available in the physics computational laboratory of SFSU.

Introduction

Studying the attitude of $a(t)$ (with $a(t_{now}) = 1, a(0) = 0$), for a *flat universe*, for the matter, radiation and cosmological constant trough the Friedman equation we see that

$$a_{radiation}(t) \propto t^{\frac{1}{2}},\ a_{matter}(t) \propto t^{\frac{2}{3}} \text{ and } a_{\Lambda}(t) \propto e^{\sqrt{\frac{8\pi G\rho_{\Lambda} t}{3}}}.$$

So we can see that in the early universe the radiation was predominant (in this case $a(t) = \sqrt{2 \cdot H_0 \cdot t\sqrt{\Omega_{R0}}}$) and the other contribution are negligible. Thanks to the blackbody distribution we find that the energy density for the radiation is $\epsilon_{radiation} = \rho_{radiation}c^2 = \alpha T^4$, but we know that $\rho_{radiation} \propto \dfrac{1}{a^4}$ so we obtain:

$$T \propto \frac{1}{a} \propto \frac{1}{t^{\frac{1}{2}}}.$$

As we know the universe is expanding and a is getting bigger as the time pass, so our last result means that the universe cools as it expands.
As the universe expands its densities change, but as far as barionic molecules and energy particles cannot be created nor destroyed their number is always the same. On the other side their densities are proportional to the factor $\dfrac{1}{a^3}$, which gets smaller with time. We denote with $N\gamma[t]$ and $NB[t]$ the photons number density and the barions number density respectively (in general we denote as $Ny[t]$ the number density of the particle/element y at the time t. Given an element y we define the *ionization fraction of y* as $Xy = \dfrac{Ny}{Nbarions}$). As both the two densities include the term a^3 at their denominator we have that their ratio will be a constant number.
At the beginning of the universe (when $a(t)$ is a very small positive number) the temperature is so high that photons are able to break the nuclear bonding energy (1 MeV) so all the protons and neutrons are separated and don't create any nucleus. But as the universe expands the photons are not so powerful anymore and the creation of nuclei begins (ionized nuclei because the photon energy is still able to break the electron bond); this process is known as *Nucleosynthesis*. Plugging into the equation $T = \dfrac{2.7}{10^9 \cdot a(t)}$ the value of

$a(t)$ and solving for t (the temperature-energy relation is easy to find thanks to the Boltzmann constant) we find that Nucleosynthesis started circa 340 seconds after the Big Bang.
When the universe is still too hot to prevent nuclei to form, protons and neutrons are in thermal equilibrium and their number density is given by $N \propto m^{\frac{3}{2}} \cdot e^{\frac{-mc^2}{k_B T}}$. Until the temperature is high the exponential factor is not important, but as the universe cools then it starts being relevant. In particular if we compute the neutrons-to-protons ratio and we assume that the universe has cooled a little bit to let the neutrons decay in protons, then we find $\frac{Nn}{Np} = EXP[\frac{-(m_n - m_p)c^2}{k_B T(= 0.8MeV)}] = \frac{1}{5}$.
When the temperature will get cooler protons and neutrons will start to combine to form the isotopes of Hydrogen and Helium. The existing reactions at this temperatures are collected in the Wagoner et alia article and shows that the following elements are being created: Deuterium, He3 (very common because of the high abundance of protons), He4 (very common because very stable element), Tritium, Lithium 7, Berillium. In our discussion we will take into consideration the ionization fraction rates of protons, neutrons, He3, He4, Tritium, Deuterium.
Computing the ratio neutrons-to-proton for this age using the previous formula we find the smaller value of $\frac{1}{7.3}$.
Thanks to Nucleosynthesis, by observing the elements abundances we are able to predict a range for the baryonic (all things made of protons and neutrons) density:

$$0.016 \leq \Omega_{Barionic}(now) \cdot h^2 \leq 0.024.$$

The predicted amount of matter in the universe is supposed to be $\Omega_M = 0.25$, then there is a huge difference between these two values, which means that most of the matter in the universe is not barionic (which is instead a small percentage). This prediction falsify also the hypotesis of the existence of huge quantities of brown stars not detectable. Definitely nucleosynthesis stands among the many hints suggesting for the existence of *dark matter*.

Method

During the Nucleosynthesis process most of the barionic elements now present in the universe formed. This process lasted for a very short period of time (relatively to the age of the universe) because the main temperature of the universe quickly cooled down and elements were further each other as the time past (then their probability to meet and form a nuclei decreased). In our calculations we will make the following hypothesis (in the parenthesis we will indicate how each parameter appears in the Mathematica code):

$$H_0 = 70\frac{Km}{s \cdot Mpc}, \ (H0);$$

$\Omega_{Radiation0} = 8 \times 10^{-5}$, $(OR0)$;

$T_g(t) = \dfrac{2.7}{10^9 \times a(t)}$, (Tg);

$\rho_{b0} = \dfrac{3 \cdot \Omega_{b0} \cdot H_0^2}{8 \cdot \pi \cdot G}$, $(rhob0)$;

$\rho_b(t) = 0.04 \cdot \rho_{critical} = 0.04 \dfrac{3 \cdot H_0^2}{8 \cdot \pi \cdot G}$, $(rhob)$;

$\Omega_{B0} = 0.04$, $(OB0)$.

To calculate the rate of Deuterium at the end of the Nucleosynthesis we need to take into consideration all the reactions in which Deuterium is involved. In truth to make a very precise analysis of the rate of Deuterium we should take into consideration all the elements that were created during this process, while we will take in account only the rates of Neutrons (n), Protons (p), Deuterium (D), Helium^3 (He3), Helium^4 (He4) and Tritium (T).This restriction won't affect significantly our study because these elements are the most commons in the universe among the barionic particles.
The first important reaction is the decay of neutrons to protons (event that happens every 882s); then we have the creation of Deuterium from the combination of one proton with one neutron (this reaction and the following are reversible because of the high temperature of the universe); but Deuterium can be also obtain as a rusult of other combinations of elements (e.g.: $p + D \leftrightarrow T + \gamma$), then we will represent only the reactions that occur among these six nuclei.
The easiest way to study the relations among the elements is to describe the rate of each of them as a differential equation. The rate will increase in time if the element is the result of a reaction involving other elements while it will decrease as the element decaying gives birth to another reaction. Thus to obtain the variation of rate of the element y, which will be the first derivative of the ionization fraction Xy[t], we will only need to add all the contributions according to the possible reactions in which the y element is involved. It is worthy to observe that now Xy[t] depends on time because during Nucleosynthesis the high temperature and density of the universe changes the rates of the elements as the time passes, which doesn't happen anymore with such an huge amount of particles nowadays (and so is neglectable).
Each direction of a reaction is characterized by a certain rate of the elements involved in the creation of the new nuclei; then when adding the contribute to the differential equations we will care not only about its sign (some rates will appear positive when creating nuclei and negative when decaying) but also of multiplying by the direction reaction rate factor. Wagoner et alia in their article provide all these necessary rates factors data, which have been calculated trough experiments, and also the following equation:

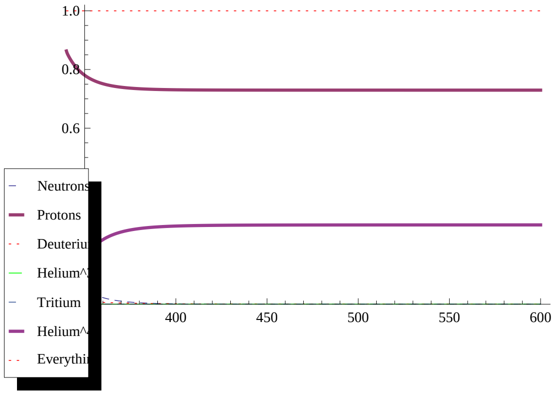

Figure 1: Rates of the elements from the beginning of Nucleosinthesis ($t = 340s$) to $t = 6000s$.

$$\frac{1}{A_l}\frac{dX_l}{dt} = \pm \sum_i \frac{X_i}{A_i}\lambda[i] \pm \sum_{i,j} \frac{X_i}{A_i}\frac{X_j}{A_j}[ij] \pm \sum_{i,j,k} \frac{X_i}{A_i}\frac{X_j}{A_j}\frac{X_k}{A_k}\lambda[i,j,k]$$

This equation describe the variation of rate of the ionization fraction of the element l, and A_i is the atomic number of the element i. If we have that in one reaction (e.g. in Wagoner reaction (7)) there is more than one atom of the same element then we will multiply to all the contributions of that reaction the number of atoms intervening.
As we write down all the six equations (as shown in Figure 4 at the end of the paper) we see that to solve our system we use the function "DSolve" and we apply initial condition (at time of 340 seconds) different from 0 only for protons and neutrons.

In order to compute the relation between Ω_{B0} and the rate of Deuterium we use the interpolation function "ListPlot" and interpolate using a polynomial of degree 3. Finally we use the function "Plot" to have a plot of the rates of the elemets from $t = 340s$ to $t = 6000s$ (we see that at this point all of the elements' rates are very stable); we also use the function "LogLogPlot" in order to have a more detailed and precise plot of how rates are going.

Conclusions

As we can observe from the first graphic (Figure 1) we imposted the equations and the initial conditions such that Xbarionic[t]=1 for all the time.

Furthermore we see that the rate of Deuterium quickly stabilize (we can also use Figure 2 to have a more precise plot) and with the current value of Ω_{B0} (0.04) we have that

$$XD[today] \approx 0.00003,$$

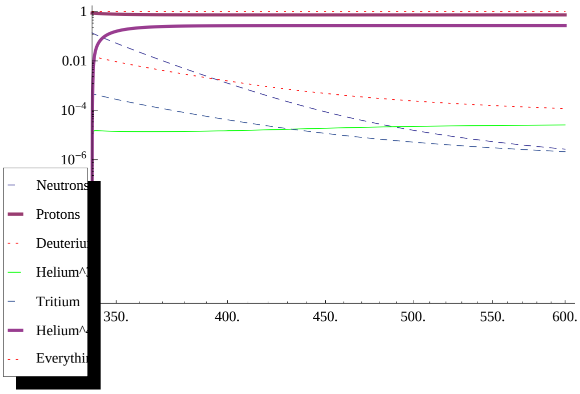

Figure 2: Rates of the elements from the beginning of Nucleosinthesis ($t = 340s$) to $t = 6000s$ with both x and y axis growing with a log rate.

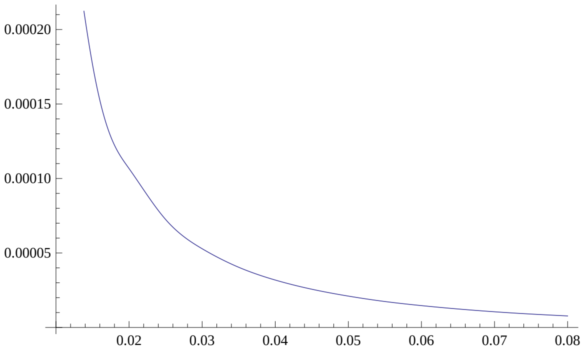

Figure 3: On the x-axis we have Ω_{B0} (which range for the possible expected today expected values [0.01,0.08]) and on the y-axis we have $XD[t = 6000]$ (with which we approximate $XD[t = today]$).

which verge on the current observed rate of Deuterium in the universe. We also provided a plot of the rates with logarithmic scale both for x and y axes. As a last observation we were also able to detect the relation standing between Ω_{B0} and the today rate of Deuterium XD[today]: in Figure 3 we plotted a interpolation of the collected data.

Then we can see that if we want the current barionic density to be higher then we have as a consequence a smaller XD[today]. But we are able to predict the amount of XD[today], which would be in contrast with such a smaller value; thus we got a proof that most of the matter in the universe is not Barionic.

```
eq1 = Xn'[t] == Xn'[t] == (* reaction 1*)
   -Xn[t] / 882 - Xn[t] * Xp[t] * (pn) + XD[t] * lambdaD / 2(* reaction 6*) -
    Xn[t] * XHe3[t] * nHe3gamma / 3 + XHe4[t] * lambdaHe4n / 4(* reaction 7*) +
    XD[t] * XD[t] * DDn / 4 - Xn[t] * XHe3[t] * nHe3D / 3(* reaction 3*) -
    Xn[t] * XD[t] * nD / 2 + XT[t] * lambdaT / 3(* reaction 4*) - Xn[t] * XHe3[t] * nHe3p / 3 +
    Xp[t] * XT[t] * pTn / 3(* reaction 11*) - Xn[t] * XHe4[t] * He4n / 4 + XD[t] * XT[t] * DT / 6
    (* reaction 13*) + 2 * XT[t] * XT[t] * TT / 9 - 2 * XHe4[t] * Xn[t] * Xn[t] * He4nn / 4
    (* reaction 14*) + XHe3[t] * XT[t] * He3Tpn / 9 - XHe4[t] * Xp[t] * Xn[t] * He4pn / 4;

eq2 = Xp'[t] == Xp'[t] == (* reaction 1*)
   Xn[t] / 882 - Xn[t] * Xp[t] * pn + XD[t] * lambdaD / 2(* reaction 2*) -
    Xp[t] * XD[t] * pD / 2 + XHe3[t] * lambdaHe3 / 3(* reaction 10*) -
    Xp[t] * XHe4[t] * He4p / 4 + XD[t] * XHe3[t] * DHe3 / 6(* reaction 12*) +
    2 * XHe3[t] * XHe3[t] * He3He3 / 9 - 2 * Xp[t] * Xp[t] * XHe4[t] * He4pp / 4
    (* reaction 4*) - Xp[t] * XT[t] * pTn / 3 + Xn[t] * XHe3[t] * nHe3p / 3
    (* reaction 5*) - Xp[t] * XT[t] * pTgamma / 3 + XHe4[t] * lambdaHe4p / 4
    (* reaction 8*) + XD[t] * XD[t] * DDp / 4 - Xp[t] * XT[t] * pTD / 3
    (* reaction 14*) + XHe3[t] * XT[t] * He3Tpn / 9 - XHe4[t] * Xp[t] * Xn[t] * He4pn / 4;

eq3 = XD'[t] / 2 == (* reaction 1*)
   Xn[t] * Xp[t] * pn - XD[t] * lambdaD / 2(* reaction 2*) - Xp[t] * XD[t] * pD / 2 +
    XHe3[t] * lambdaHe3 / 3(* reaction 7*) - XD[t] * XD[t] * DDn (2 / 4) +
    2 * Xn[t] * XHe3[t] * nHe3D (2 / 3)(* reaction 9*) - 2 * XD[t] * XD[t] * DDgamma / 4 +
    2 * XHe4[t] * lambdaHe4D / 4(* reaction 10*) + Xp[t] * XHe4[t] * He4p / 4 -
    XD[t] * XHe3[t] * DHe3 / 6(* reaction 3*) - Xn[t] * XD[t] * nD / 2 + XT[t] * lambdaT / 3
    (* reaction 8*) - 2 * XD[t] * XD[t] * DDp / 4 + 2 * Xp[t] * XT[t] * pTD / 3
    (* reaction 11*) + Xn[t] * XHe4[t] * He4n / 4 - XD[t] * XT[t] * DT / 6
    (* reaction 15*) + XHe3[t] * XT[t] * He3TD / 9 - XHe4[t] * XD[t] * He4D / 8;

sol = NDSolve[{eq1, eq2, eq3, eq4, eq5, eq6, Xn[340] == 0.1369,
    Xp[340] == 0.8631, XD[340] == 0, XHe3[340] == 0, XT[340] == 0, XHe4[340] == 0},
   {Xn[t], Xp[t], XD[t], XHe3[t], XT[t], XHe4[t]}, {t, 340, 6000}];

Plot[{Xn[t] /. sol, Xp[t] /. sol, XD[t] /. sol, XHe3[t] /. sol, XT[t] /. sol, XHe4[t] /. sol,
  Xn[t] + Xp[t] + XD[t] + XHe3[t] + XT[t] + XHe4[t] /. sol}, {t, 340, 600}, PlotLegend →
  {"Neutrons", "Protons", "Deuterium", "Helium^3", "Tritium", "Helium^4", "Everything"},
 PlotStyle → {Dashed, Thick, {Dotted, Red}, {Green}}]

LogLogPlot[
 {Xn[t] /. sol, Xp[t] /. sol, XD[t] /. sol, XHe3[t] /. sol, XT[t] /. sol, XHe4[t] /. sol,
  Xn[t] + Xp[t] + XD[t] + XHe3[t] + XT[t] + XHe4[t] /. sol}, {t, 340, 600}, PlotLegend →
  {"Neutrons", "Protons", "Deuterium", "Helium^3", "Tritium", "Helium^4", "Everything"},
 PlotStyle → {Dashed, Thick, {Dotted, Red}, {Green}}]

ListPlot[{{0.01, 0.000362255}, {0.015, 0.000176508},
  {0.02, 0.000106655}, {0.025, 0.0000724955}, {0.03, 0.0000528472},
  {0.035, 0.0000404091}, {0.04, 0.0000318338}, {0.045, 0.0000256704},
  {0.05, 0.0000210532}, {0.055, 0.0000174191}, {0.06, 0.0000146694},
  {0.065, 0.0000124073}, {0.07, 0.0000105293}, {0.075, 8.99624 * (10^(-6))},
  {0.08, 7.73553 * (10^(-6))}}, Joined → True, InterpolationOrder → 3]
```


Figure 4: This is the code used to calculate the rate of the universe. We are showing only the rates for Protons (Xp[t]),Neutrons (Xn[t]) and Deuterium (XD[t]).

References

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[3] Ken Croswell, *The Alchemy of the Heavens*, Random House Publishers 1996.

[4] Robert V. Wagoner, *On the synthesis of elements at very high temperatures*, Caltech, Pasadena, California, 1966.