TD L1 BST (GROUPE 1) - 23/10/2025

Informations : Tous les fichiers peuvent être trouvés à l'adresse

https://poisson.phc.dm.unipi.it/~dinunzio/Teaching

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Rappels :

- $\int e^x \, dx = e^x$

Si $a \in \mathbb{R}$, $a \neq 0$, alors

- $\int e^{ax} \, dx = \frac{1}{a} e^{ax}$

- $\int \sin(x) \, dx = -\cos(x)$

- $\int \cos(x) \, dx = \sin(x)$

Si $a \in \mathbb{R}$, $a \neq -1$, alors

- $\int x^a \, dx = \frac{x^{a+1}}{a+1}$.

Si $a = -1$

$\int \frac{1}{x} \, dx = \ln(|x|)$

INTEGRATION PAR PARTIES

Soient $f(x), g(x)$ deux fonctions dérivables. Alors

$$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$f'(x)\cdot g(x) = (f(x)\cdot g(x))' - f(x)\cdot g'(x)$$

$$\int f'(x)\, g(x)\, dx = \int \left((f(x)\cdot g(x))' - f(x)\cdot g'(x)\right) dx$$

$$= \int (f(x)\cdot g(x))'\, dx - \int f(x)\cdot g'(x)\, dx$$

$$= f(x)\cdot g(x) - \int f(x)\cdot g'(x)\, dx$$

On a donc trouvé :

$$\boxed{\int f'(x)\cdot g(x)\, dx = f(x)\cdot g(x) - \int f(x)\cdot g'(x)\, dx}$$

Formule d'intégration par parties

Ex. 1) $\int \underbrace{x}_{g(x)} \cdot \underbrace{e^x}_{f'(x)} \, dx$

$f(x) = \int e^x dx = e^x$

$g'(x) = 1$

$$\int \underbrace{x}_{g(x)} \cdot \underbrace{e^x}_{f'(x)} \, dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x) \, dx$$

$$= e^x \cdot x - \int e^x \cdot 1 \, dx$$

$$= e^x \cdot x - \int e^x \, dx$$

$$= e^x \cdot x - e^x \quad = e^x (x-1)$$

→ Comment faire le choix de f' et g ?

$$\int \underbrace{x}_{f'(x)} \cdot \underbrace{e^x}_{g(x)} \, dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x) \, dx$$

$$= \frac{x^2}{2} e^x - \underbrace{\int \frac{x^2}{2} e^x \, dx}_{\text{pas du tout plus facile !}}$$

Donc le choix $f'(x) = e^x$ et $g(x) = x$ été meilleur.

2) $\int \underbrace{x}_{} \cdot \underbrace{\cos(x)}_{} \, dx$

NON → f' $\quad g$ mais $f(x) = \frac{x^2}{2}$

$g \quad f'$ $\qquad g'(x) = -\sin(x)$

Je trouverais $\int f \cdot g' \, dx = \int \frac{x^2}{2} (-\sin x) dx$

Donc il faut considérer $f'(x) = \cos(x) \leadsto f(x) = \sin(x)$

$g(x) = x \leadsto g'(x) = 1$

$\leadsto$ je trouverais (dans la formule d'intég. par parties)

$$\int f(x) \cdot g'(x)\,dx = \int \sin(x) \cdot 1 \, dx$$

Beaucoup plus facile de $\int x \cos(x)\,dx$!

Donc :

$$\int \underbrace{x}_{g} \cdot \underbrace{\cos(x)}_{f'}\,dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x)\,dx$$

$$= \sin(x) \cdot x - \int \sin(x) \cdot 1 \, dx$$

$$= \sin(x) \cdot x - \int \sin(x)\,dx$$

$$= \sin(x) \cdot x - (-\cos(x))$$

$$= \sin(x) \cdot x + \cos(x).$$

3) $$\int \underbrace{x}_{f'} \cdot \underbrace{\ln(x)}_{g}\,dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x)\,dx$$

$f(x) = \frac{x^2}{2}$

$g'(x) = \frac{1}{x}$

$$= \frac{x^2}{2} \cdot \ln(x) - \int \frac{x^2}{2} \cdot \frac{1}{x}\,dx$$

$$= \frac{x^2}{2} \ln(x) - \int \frac{x}{2}\,dx$$

$$= \frac{x^2}{2} \ln(x) - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln(x) - \overbrace{\frac{x^2}{4}}^{\frac{1}{2} \cdot \frac{1}{2} x^2}$$

4) $\int \underbrace{x^3}_{f'} \underbrace{\ln(x)}_{g} \, dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x) \, dx$

$f(x) = \frac{1}{4} x^4$

$g'(x) = \frac{1}{x}$

$= \frac{1}{4} x^4 \cdot \ln(x) - \int \underbrace{\frac{1}{4} x^4 \cdot \frac{1}{x}}_{= \frac{x^4}{x} = x^3} \, dx$

$= \frac{1}{4} x^4 \ln(x) - \frac{1}{4} \underbrace{\int x^3 \, dx}_{\frac{x^4}{4}}$

$= \frac{1}{4} x^4 \ln(x) - \frac{1}{16} x^4$

5) $\int \ln(x) \, dx = \int \underbrace{1}_{f'} \cdot \underbrace{\ln(x)}_{g} \, dx$

$f(x) = x$

$g'(x) = \frac{1}{x}$

$= f(x) \cdot g(x) - \int f(x) \cdot g'(x) \, dx$

$= x \cdot \ln(x) - \int \underbrace{x \cdot \frac{1}{x}}_{\frac{x}{x} = 1} \, dx$

$= x \cdot \ln(x) - \int 1 \, dx$

$= x \cdot \ln(x) - x$.

Exercice 32. Calculer, grâce à le méthode d'intégration par partie, les primitives suivantes :

a. $\int xe^x\,dx$

b. $\int x\ln(x)\,dx$

c. $\int (x+1)\sin(x)\,dx$

d. $\boxed{\int e^x\sin(x)\,dx}$

e. $\int \ln(x)\,dx$

f. $\int x^2\sin(x)\,dx$

g. $\int x^2e^x\,dx$

h. ~~$\int \frac{x}{(\cos(x))^2}\,dx$~~

i. $\int xe^{3x}\,dx$

(Ind. pour i. utiliser que, pour $a\in\mathbb{R}$ constant, $\int e^{ax}dx = \frac{1}{a}e^{ax} + \cancel{C}$).

$a \neq 0$

Pour d. : il est plus difficile

Sol.

a. Déjà vu (Ex. 1))

b. Déjà vu (Ex. 3))

c. $\int \underbrace{(x+1)}_{g(x)}\underbrace{\sin(x)}_{f'(x)}dx = \underbrace{-\cos(x)}_{F(x)}\cdot\underbrace{(x+1)}_{g(x)} - \int \underbrace{(-\cos(x))}_{F(x)}\cdot\underbrace{1}_{g'(x)}\,dx$

$F(x) = -\cos(x)$
$g'(x) = 1$

$= -\cos(x)\cdot(x+1) + \int \cos(x)\,dx$

$= -\cos(x)\cdot(x+1) + \sin(x)$

d. Après...

e. Déjà vu (Ex. 5))

f. $\int \underbrace{x^2}_{g(x)}\underbrace{\sin(x)}_{f'(x)}dx = \underbrace{-\cos(x)}_{F}\cdot\underbrace{x^2}_{g} - \int \underbrace{(-\cos(x))}_{F}\cdot\underbrace{2x}_{g'}\,dx$

$F(x) = -\cos(x)$
$g'(x) = 2x$

$= -\cos(x)\cdot x^2 + \underbrace{2\int x\cdot\cos x\,dx}_{\text{Déjà vu (Ex. 2))}}$

Mais quand-même

$$= -\cos(x)\cdot x^2 + 2\int \underbrace{x}_{g}\cdot\underbrace{\cos(x)}_{f'}\,dx$$

$f(x) = \sin(x)$
$g'(x) = 1$

$$= -\cos(x)\cdot x^2 + 2\left(\underbrace{\sin(x)}_{f}\cdot\underbrace{x}_{g} - \int \underbrace{\sin(x)}_{f}\cdot\underbrace{1}_{g'}\,dx\right)$$

$$= -\cos(x)\cdot x^2 + 2\left(\sin(x)\cdot x + \cos(x)\right)$$

$\int \sin(x)\,dx = -\cos(x)$
$-$ '' '' $= +\cos(x)$

$$= (-\cos x)\, x^2 + (2\sin x)\, x + 2\cos(x).$$

g. $$\int \underbrace{x^2}_{g}\underbrace{e^x}_{f'}\,dx = e^x\cdot x^2 - \int 2x\,e^x\,dx$$

$f(x) = e^x$
$g'(x) = 2x$

$$= e^x\cdot x^2 - 2\underbrace{\int x\,e^x\,dx}_{\text{Déjà vu (Ex. 1)}}$$

$$= e^x\cdot x^2 - 2\int \underbrace{x}_{g}\cdot\underbrace{e^x}_{f'}\,dx$$

$f(x) = e^x$
$g'(x) = 1$

$$= e^x\cdot x^2 - 2\left(\underbrace{e^x}_{f}\cdot\underbrace{x}_{g} - \int \underbrace{e^x}_{f}\cdot\underbrace{1}_{g'}\,dx\right)$$

$$= e^x\cdot x^2 - 2e^x\cdot x + 2e^x.$$

i. $$\int \underbrace{x}_{g} \underbrace{e^{3x}}_{f'} dx = \frac{1}{3} e^{3x} \cdot x - \int \frac{1}{3} e^{3x} \cdot 1 \, dx$$

$f(x) = \frac{1}{3} e^{3x}$

$g'(x) = 1$

$$= \frac{1}{3} e^{3x} \cdot x - \frac{1}{3} \int e^{3x} dx$$

$$= \frac{1}{3} e^{3x} \cdot x - \frac{1}{3} \left(\frac{1}{3} e^{3x} \right)$$

$$= \frac{1}{3} e^{3x} \cdot x - \frac{1}{9} e^{3x} .$$

d. $$\int \underbrace{e^x}_{f'} \cdot \underbrace{\sin(x)}_{g} dx = e^x \cdot \sin(x) - \int \underbrace{e^x}_{f'} \cdot \underbrace{\cos(x)}_{g} dx$$

$f(x) = e^x$

$g'(x) = \cos(x)$

$f(x) = e^x$

$g'(x) = -\sin(x)$

$$= e^x \cdot \sin(x) - \left(\overbrace{e^x}^{f} \overbrace{\cos(x)}^{g} - \int \overbrace{e^x}^{f} \overbrace{(-\sin(x))}^{g'} dx \right)$$

$$= e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x) \, dx$$

On appelle $F(x) = \int e^x \sin(x) \, dx$. Alors, on a trouvé :

$$F(x) = e^x \sin(x) - e^x \cos(x) - F(x)$$

$$2F(x) = e^x \sin(x) - e^x \cos(x)$$

$$\Rightarrow \int e^x \sin(x) \, dx = F(x) = \frac{1}{2} \left(e^x \sin(x) - e^x \cos(x) \right) .$$