EXERCISES OF WEEK THREE

Exercise 1. Given the two lines

$$\ell := \ell(P, v), \quad \ell' := \ell(Q, w)$$

find the intersection point, where

$$P = (1, 0), \quad v = (3, 4), \quad Q = (2, 1), \quad w = (0, 2)$$

Then, evaluate the distance

$$\mathrm{dist}(R, \ell)$$

where $R = (2, 3)$.

Solution. We have

$$\ell = \{(1 + 3t, 4t) \mid t \in \mathbb{R}\}, \quad \ell' = \{(2, 1 + 2t) \mid t \in \mathbb{R}\}.$$

We use the intersection formula which involves the cross product:

$$T = P + \left(\frac{\overrightarrow{PQ} \times w}{v \times w} \right) v.$$

We have

$$\overrightarrow{PQ} = (1, 1), \quad \overrightarrow{PQ} \times w = 2, \quad v \times w = 6.$$

Then

$$T = P(1, 0) + (1, 4/3) = (2, 4/3).$$

In order to evaluate the distance $d(R, \ell)$, we use the formula

$$\mathrm{dist}(R, \ell) = \frac{|\overrightarrow{PQ} \times v|}{|v \times w|} = \frac{1}{6}.$$

□

Exercise 2. Find the parametric form of the line which contains the points

$$P_1 = (1, 3), \quad P_2 = (2, 7).$$

Find the parametric form and the normal form of the plane containing the three points

$$P = (1, 0, 1), \quad Q = (2, -1, 3), \quad R = (1, 0, 0);$$

find the parametric form and the normal form of the plane containing the following point and line (as a subset)

$$P = (1, 0, 1), \quad \ell(Q, v)$$

where

$$Q = (0, 0, 0), \quad v = (1, 1, 1).$$

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Solution. The parametric form of the first line is

$$\ell(P_1, \overrightarrow{P_2P_1}) = \{(1+t, 3+4t) \mid t \in \mathbb{R}\}.$$

In order to find the parametric form of the plane, we evaluate

$$\overrightarrow{PQ} = (1, -1, 3), \quad \overrightarrow{PR} = (0, 0, -1).$$

The parametric form of the plane is

$$\pi(P, \overrightarrow{PQ}, \overrightarrow{PR}) = \{(1+t, -t, 1+3t-s) \mid t, s \in \mathbb{R}\}.$$

For the normal form, we have

$$\overrightarrow{PQ} \times \overrightarrow{PR} = (1, -1, 3) \times (0, 0, -1) = (1, 1, 0).$$

Then

$$\pi(P, \overrightarrow{PQ} \times \overrightarrow{PR}) : x - 1 + y = 0.$$

The parametric form of the plane containing the line ℓ and the point Q is

$$\pi(P, \overrightarrow{PQ}, v) = \{(1-t, s, 1-t) \mid t, s \in \mathbb{R}\};$$

for the normal form, we have

$$\overrightarrow{PQ} \times v = (1, 0, 1) \times (1, 1, 1) = (-1, 0, 1).$$

Then the normal form is

$$\pi(P, \overrightarrow{PQ} \times v) : -(x-1) + z - 1) = z - x = 0.$$

□

Exercise 3. Given $v, w, z \in E^3$, show that

$$\begin{vmatrix} v_1 & w_1 & z_1 \\ v_2 & w_2 & z_2 \\ v_3 & w_3 & z_3 \end{vmatrix} = (v \times w) \cdot z.$$

Solution. In order to compute the determinant of A we use the Laplace method with respect to the last column

$$\begin{aligned} \det(A) &= z_1 \begin{vmatrix} v_2 & w_2 \\ v_3 & w_3 \end{vmatrix} - z_2 \begin{vmatrix} v_1 & w_1 \\ v_3 & w_3 \end{vmatrix} + z_3 \begin{vmatrix} v_1 & w_1 \\ v_2 & w_2 \end{vmatrix} \\ &= z_1(v_2w_3 - v_3w_2) - z_2(v_1w_3 - v_3w_1) + z_3(v_1w_2 - v_2w_1) \\ &= z_1(v \times w)_1 + z_2(v \times w)_2 + z_3(v \times w)_3 = (v \times w) \cdot z. \end{aligned}$$

□