1. Applied vectors of the euclidean space

Given a natural number n, we consider the set

$$\mathbb{R}^n := \{(x_1, x_2, \ldots, x_n) \mid x_i \in \mathbb{R}\, \forall i\}.$$

It is known that such set has a linear structure defined as

$$(v + w)_i := v_i + w_i \quad (\lambda v)_i = \lambda v_i$$

for every $v, w \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$.

We wish to make a distinction between the set $\mathbb{R}^n$ and the linear space

$$E := (\mathbb{R}^n, +).$$

We will use the notation $P, Q, R \in \mathbb{R}^n$ for points and $v, w, z \in E$ for vectors.

We consider the following product space

$$E \times \mathbb{R}^n = \{(P, v) \mid P \in \mathbb{R}^n,\, v \in E\}.$$

The idea of such representation comes mainly from problems of Physics: the fact that a force F is applied at a point P, is represented by the pair

$$(P, F).$$

Definition 1. We call the elements of $E \times \mathbb{R}^n$ *applied vectors* and $E \times \mathbb{R}^n$ *space of the applied vectors*. In (v, P), we call P *initial point* and v *displacement*.

In order to stress the distinction between vector and points, we will sometimes use the notation

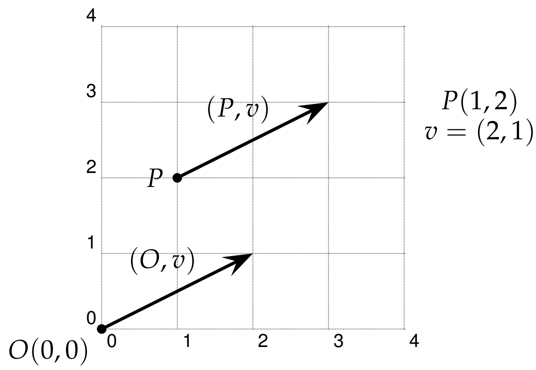
$$P(x_1, x_2, \ldots, x_n)$$

for points and

$$v := (v_1, v_2, \ldots, v_n)$$

for vectors.

A useful graphical representation of an applied vector (v, P) is in the use of arrows as in the following picture



Definition 2. Given two points

$$P(x_1, x_2, \ldots, x_n), \quad Q(y_1, y_2, \ldots, y_n)$$

we can define the *displacement between* P *and* Q

$$\overrightarrow{PQ} := (y_1 - x_1, y_2 - x_2, \ldots, x_n - y_n).$$

We also define the point $P + v$ of coordinates

$$(x_1 + v_1, x_2 + v_2, \ldots, x_n + v_n).$$

We call it *endpoint* of the applied vector (P, v).

Another notation for the displacement is $Q - P$. According to definition , the initial point of $(P, \overrightarrow{PQ})$ is P and its displacement is $\overrightarrow{PQ}$.

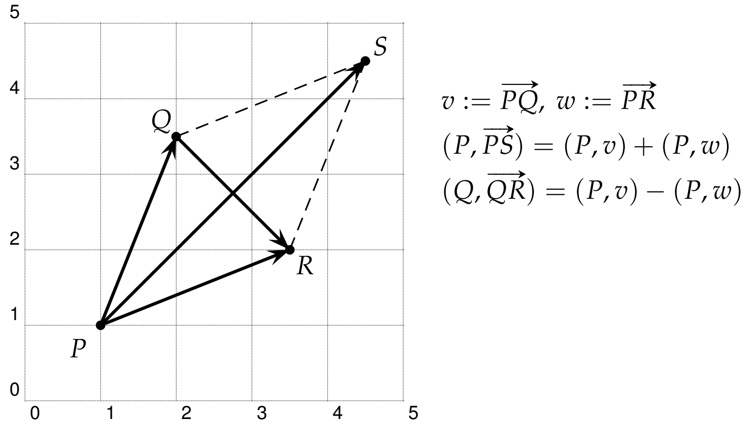
Definition 3. Given applied vectors (P, v) and (P, w), we define the sum

$$(P, v) + (P, w) := (P, v + w)$$

and the difference

$$(P, v) - (P, w) := (P, v - w).$$

We give a graphic representation of the sum and the difference in the picture below.



The following properties can be checked easily from the previous definitions:

Proposition 1. *For every $P, Q, R \in \mathbb{R}^n$*

$$P + \overrightarrow{PQ} = Q$$
$$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$$
$$(P + v) + w = P + (v + w).$$

In terms of the graphical representation of applied vectors, the second equality have the geometric interpretation given in the diagram