

EXERCISES OF WEEK TWO

Exercise 1 ((c), ex. 4, EXERCISES 1.1, page 25 of [Pin71]). If the sentence is true, show it with a truth table. If it is false, give an example. For instance

$$P \vee Q \Rightarrow Q$$

is false when P is true and Q is false, $P \vee Q$ is true and Q is false.

(a) $P \wedge Q \Rightarrow Q$

(d) $(P \Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$

(b) $Q \Rightarrow P \vee Q$

(c) $Q \Rightarrow P \wedge Q$

Solution.

(a)

| P | Q | $P \wedge Q$ | $P \wedge Q \Rightarrow Q$ |
|-----|-----|--------------|----------------------------|
| T | T | T | T |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |

(b)

| P | Q | $P \vee Q$ | $Q \Rightarrow P \vee Q$ |
|-----|-----|------------|--------------------------|
| T | T | T | T |
| T | F | T | T |
| F | T | T | T |
| F | F | F | T |

(c) it is false; for example, if Q is true and P is false, then $P \wedge Q$ is false. So $Q \Rightarrow P \wedge Q$ is false

(d)

| P | Q | $\neg P$ | $\neg Q$ | $P \Rightarrow Q$ | $\neg Q \Rightarrow \neg P$ | $(P \Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$ |
|-----|-----|----------|----------|-------------------|-----------------------------|---|
| T | T | F | F | T | T | T |
| T | F | F | T | F | F | T |
| F | T | T | F | T | T | T |
| F | F | T | T | T | T | T |

□

Exercise 2. In the following

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| | X | Y | Z | T |
|---|---|---|---|---|
| X | 0 | 0 | 0 | 0 |
| Y | 1 | 0 | 0 | 1 |
| Z | 0 | 1 | 0 | 1 |
| T | 0 | 0 | 0 | 0 |

What are

- (a) sets
- (b) proper classes
- (c) the class $\{x \mid x \notin x\}$?

Solution.

- (a) Y, Z
- (b) X, T
- (c) T.

□

REFERENCES

Pin71. Charles C. Pinter. *Set theory*. Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1971.