M comp. man..

Def.: $\Gamma < \mathrm{Aut}(M)$ is:

1) free if Γ has no fixed pts;
2) properly discontinuous if $K_1, K_2 \subseteq M$ cpt $\Rightarrow$ $\Rightarrow \{g \in \Gamma \mid g(K_1) \cap K_2 \neq \emptyset\}$ is finite.

Thm.: if Γ acts freely and prop. disc. on M, then M/Γ is a comp. man..

Proof: M conn., paracpt $\Rightarrow M$ countable union of cpt sets.

$M^* = M/\Gamma$ as a set, p^* = orbit of p.

We want to show that, given $q \in M$, $\exists$ neigh. $U \ni q$ s.t. if $p_1, p_2 \in U$, $p_1 \neq p_2$ then $p_1^* \neq p_2^*$. In fact, $\exists\, U$ s.t. $g(U) \cap U = \emptyset$ $\forall g \in \Gamma \setminus \{id\}$.

M locally cpt $\Rightarrow \exists$ basis $U_1 \supsetneq U_2 \supsetneq \dots$ of relatively cpt neighs. of $q \Rightarrow F_m = \{g \mid g(U_m) \cap U_m \neq \emptyset\}$ is finite. $F_1 \supseteq F_2 \supseteq \dots$; therefore, if $\exists\ id \neq g \in F_m\ \forall m \geq m_0$, then $g(U_m) \cap U_m \neq \emptyset$ and U_m shrinks to $q \Rightarrow g(q) = q$, contradiction. So $\exists\, U$ as claimed above.

$p_1, p_2 \in U$, $p_1 \neq p_2 \Rightarrow p_1^* \neq p_1^*$, so (shrinking U if necessary) we get a coord. system $(z_1, \dots, z_m)$ on $U \Rightarrow$ $\Rightarrow$ gives $(z_1^*, \dots, z_m^*)$ on U^*. $\square$

Ex.: $\mathbb{C}^g/\Lambda$.

Ex.: $M = \{[x_0 : \dots : x_3] \in \mathbb{P}^3 \mid x_0^5 + \dots + x_3^5 = 0\}$,
$\Gamma = \{\gamma^m \mid m = 0, \dots, 4\}$, $\gamma([x_0 : \dots : x_3]) = [\rho x_0 : \rho^2 x_1 : \rho^3 x_2 : \rho^4 x_3]$,
$\rho = e^{2\pi i/5}$.

Ex.: weighted projective space. Let $q_0, \dots, q_N$ be positive integers with gcd$=1$. We act on $\mathbb{P}^N$ by $\mathbb{Z}/(q_0) \times \dots \times \mathbb{Z}/(q_N)$. Note: $\mathbb{P}(kq_0, \dots, kq_N) \cong \mathbb{P}(q_0, \dots, q_N)$.

$\mathbb{P}[1,2,3]$, $X = \{x_0^6 + x_1^3 + x_2^2 = 0\}$.

$\pi: X \longrightarrow \mathbb{P}^1$ with $[x_0 : x_1 : x_2] \mapsto [x_0^2 : x_1]$. Calculations $\Rightarrow$ it's a 2:1 cover of $\mathbb{P}^1$ 4 branched pts $\Rightarrow X$ is a genus 1 curve.

Singular locus of $\mathbb{P}[q_0, \dots, q_N]$. If p is a prime,
$S_p = \{[x_0 : \dots : x_N] \mid x_j = 0 \text{ if } p \nmid q_j\}$, $\mathrm{Sing}\, \mathbb{P}(q_0, \dots, q_N) = \bigcup_{p \text{ prime}} S_p$.

Ex.: $\mathbb{P}[1,2,2,5]$: $S_2 = \{[0 : x_1 : x_2 : 0]\}$, $S_5 = \{[0:0:0:1]\}$.

Ex.: $\mathbb{P}[1,1,2,5]$: $S_2 = \{[0:0:1:0]\}$, $S_5 = \{[0:0:0:1]\}$.

Ex.: $f(x_0, \dots, x_N)$ homogeneous of degree $2d$ in $\mathbb{P}^N$ (smooth).
$\dim V(f) = N-1$. $\mathbb{P}[\underbrace{1, \dots, 1}_{N+1} : d]$, $y_{N+1}^2 = f(x_0, \dots, x_N)$.
$V(y^2 - f)$ is a 2:1 cover of $\mathbb{P}^N$ branched on $V(f)$.

Hodge theory works well for weighted projective varieties.

$L \to X$ hol. line bundle. What is the divisor of L?

Assume $L \to X$ has a meromorphic section $\sigma: X \to L$.

Given $p \in X$ $\exists$ a local nonvanishing hol. section s of L defined on a neigh. of $p \Rightarrow \sigma = fs$, f mero.; $\mathrm{div}(\sigma) = \mathrm{div}(f)$ in the neigh..

Case X curve. Let $\sigma_1, \sigma_2 : X \to L$ be mero. sections of $L \Rightarrow$ $\Rightarrow \sigma_1/\sigma_2$ is a mero. section of L.

If $L \to X$ has a mero. section then $\mathrm{div}(\sigma)$ modulo $\mathrm{div}(X)$ is well def., where $\mathrm{div}(X) = \{\mathrm{div}(f) \mid f: X \to \mathbb{P}^1 \text{ mero.}\}$.