$L \to \mathbb{C}/\Lambda$ hol. line bundle, σ mero. global section of f.

$\operatorname{div}(L) = \operatorname{div}(\sigma)$ is not well def.; it is modulo divisors of mero. functions.

We know them from Abel's theorem.

⌈Degree-genus formula: $C = V(f)$ smooth, f homogeneous of degree d on $\mathbb{P}^2 \Rightarrow$

$$\Rightarrow g(C) = \frac{(d-1)(d-2)}{2}. \rfloor$$

Hodge-Riemann bilinear relations for Riemann surfaces

X Riemann surface. $H^1_{DR}(X;\mathbb{C}) \cong \underbrace{H^{1,0}(X)}_{\text{hol. 1-forms}} \oplus \underbrace{H^{0,1}(X)}_{\text{anti-hol. 1-forms}}$.

M cpt ori. man., $H^k_{DR}(M;\mathbb{R}) \cong \{\alpha \in \Lambda^k(M) \mid \Delta\alpha = 0\}$,

$\Delta = dd^* + d^*d$, $d : \Lambda^k(M) \to \Lambda^{k+1}(M)$, $d^* : \Lambda^{k+1}(M) \to \Lambda^k(M)$.

$\Delta\alpha = 0 \Rightarrow \langle \Delta\alpha, \alpha\rangle = 0 \Rightarrow \langle dd^*\alpha, \alpha\rangle + \langle d^*d\alpha, \alpha\rangle = 0 \Rightarrow$

$\Rightarrow \langle d^*\alpha, d^*\alpha\rangle + \langle d\alpha, d\alpha\rangle = 0 \Rightarrow d\alpha, d^*\alpha = 0$.

Cpt Kähler: $\Box_{\bar\partial} = \bar\partial^*\bar\partial + \bar\partial\bar\partial^*$ is a scalar multiple of Δ,

$\Box_{\partial} = \partial\partial^* + \partial^*\partial$ " " " " " Δ.

$\overline{H^{1,0}} = H^{0,1}$. $\alpha \in H^{1,0}(X)\setminus\{0\} \Rightarrow$ locally $\alpha = f\,dz$, $f \not\equiv 0$.

$\alpha\wedge\bar\alpha = |f|^2 dz\wedge d\bar z = |f|^2(-2i\,dx\wedge dy) \Rightarrow \int_X i\,\alpha\wedge\bar\alpha > 0$.

$$\begin{aligned} Q : H^1(X;\mathbb{C}) \otimes H^1(X;\mathbb{C}) &\to \mathbb{C}, \\ \alpha\otimes\beta &\mapsto \int_X \alpha\wedge\beta \end{aligned}$$

$C : H^1(X;\mathbb{C}) \to H^1(X;\mathbb{C})$, $C_{|H^{1,0}} = i\,\cdot$, $C_{|H^{0,1}} = -i\,\cdot$.

H.-R. bilinear relations: $\langle\alpha,\beta\rangle = Q(C\alpha, \bar\beta)$ is a positive def. hermitian form which makes $H^{1,0}(X) \oplus H^{0,1}(X)$ orthogonal.