<u>Morphism</u>: $f: H \to H'$ is a morphism of Hodge structures if
$f(H_{\mathbb{Z}}) \subseteq H'_{\mathbb{Z}}$ and $f(F^p H_{\mathbb{C}}) \subseteq F^p H'_{\mathbb{C}}$.

Lemma: morphisms of Hodge structures are strict:
$f(F^p H_{\mathbb{C}}) = F^p H'_{\mathbb{C}} \cap f(H_{\mathbb{C}})$.

Proof: $\subseteq$: by definition. $\supseteq$: suppose $\beta \in F^p H'_{\mathbb{C}} \cap f(H_{\mathbb{C}})$, so
$\beta = f(\alpha)$, $\alpha \in H_{\mathbb{C}} \underset{\text{weight } w}{\Rightarrow} \alpha = \sum_{l} \alpha^{l, w-l} \Rightarrow$

$\Rightarrow f(\alpha) = \sum_{l} f(\alpha^{l,w-l})$. $\alpha^{l,w-l} \in F^l H_{\mathbb{C}} \cap \overline{F^{w-l} H_{\mathbb{C}}} \Rightarrow$
$\underset{f \text{ morphism}}{\Rightarrow} f(\alpha^{l,w-l}) \in F^l H'_{\mathbb{C}} \cap \overline{F^{w-l} H'_{\mathbb{C}}} = (H')^{l,w-l} \subseteq F^l H'_{\mathbb{C}} \setminus F^{l+1} H'_{\mathbb{C}}$.
So $\beta = \sum_{l} \underbrace{f(\alpha^{l,w-l})}_{\cap \, F^l H'_{\mathbb{C}} \setminus F^{l+1} H'_{\mathbb{C}}}$, $\beta \in F^p H'_{\mathbb{C}} \Rightarrow$

$\Rightarrow f(\alpha^{l,w-l}) = 0 \ \forall \, l < p$. $\square$

$X = \overline{X} \setminus S$, $\overline{X}$ smooth proj. curve, S finite set of pts, $X \hookrightarrow \overline{X}$.

$\underset{0}{\overset{\parallel}{H^1(|S|)}} \to H^1(\overline{X}) \hookrightarrow H^1(X) \xrightarrow{\text{Res}} H^0(|S|) \xrightarrow{} H^0(\overline{X})$. it is 0 on $\operatorname{Im} \text{Res}$

$\ulcorner f: \overline{X} \setminus \{p,q\}$ harmonic, local coord. s s.t. $s(p)=0$ and t s.t. $t(q)=0$. $f + \log|s|^2$, $f - \log|t|^2$ have harm. extension at p, q. $df + i * df$ is holo. $\lrcorner$

So $0 \to H^1(\overline{X}) \hookrightarrow H^1(X) \xrightarrow{\text{Res}} H^0_{\#}(|S|) \to 0$.
$0 \to H^1(\overline{X}) \hookrightarrow H^1(X) \xrightarrow{\text{Res}} H^0_{\#}(|S|) \otimes \mathbb{Z}(-1) \to 0$

$\ulcorner \mathbb{Z}(p)$ is $\mathbb{Z}$ with a Hodge structure of type $(-1,-1)$ and lattice $(2\pi i)^p \mathbb{Z}$, so $\mathbb{Z}(-1)$ is of type $(1,1)$ with lattice $\frac{1}{2\pi i}\mathbb{Z} \lrcorner$; this is exact in mixed Hodge structures.

$W_0 H^1(X) = 0$, $W_1 H^1(X) = H^1(\overline{X})$, $W_2 H^1(X) = H^1(X)$.

W increasing filtration, $\operatorname{Gr}^W_k = \dfrac{W_k}{W_{k-1}} \rightsquigarrow$
$\rightsquigarrow \operatorname{Gr}^W_1 H^1(X) \cong H^1(\overline{X})$, $\operatorname{Gr}^W_1 H^1(X) \cong H^0_{\#}(|S|) \otimes \mathbb{Z}(-1)$.