Ex.: H, H' of (pure) weight $k \Rightarrow H \oplus H'$ of (pure) weight k.

Ex.: H, H' of (pure) weight k and $l \Rightarrow H \otimes H'$ of (pure) weight $k+l$ and $(H \otimes H')^{p,q} = \bigoplus_{\substack{a+a'=p,\\ b+b'=q}} H^{a,b} \otimes (H')^{a',b'}$.

X, Y cpt Kähler man. $\Rightarrow H^m(\overbrace{X \times Y}^{\text{cpt Kähler}}) = \bigoplus_{k+l=m} H^k(X) \otimes H^l(Y)$.

Ex.: H of (pure) weight $k \Rightarrow H^*$ of pure weight $-k$, $(H^*)^{-p,-q} = \mathrm{Ann}\left(\bigoplus_{(a,b)\neq(p,q)} H^{a,b}\right)$.

Ex.: X cpt Kähler $\Rightarrow H_k(X;\mathbb{Z})$ is pure and equal to $H^k(X,\mathbb{Z})^*$ (weight k).

Ex.: $Y \subseteq X$ smooth subvar. of codim. $p \Rightarrow cl(Y) \in H^{2p}(X,\mathbb{Z})$ is pure of type (p,p). Y smooth $\Rightarrow \int_Y \alpha = \int_X \alpha \wedge cl(Y)$, $\alpha \in H^{2\dim_{\mathbb{C}} X - 2p}$. If α has more than $\underbrace{\dim_{\mathbb{C}} X - p}_{d}$ dzs then $\int_Y \alpha = 0$. So $\int_Y$ annihilates $F^{d-p+1}H^{2d-2p}(Y,\mathbb{C}) \Rightarrow cl(Y) \in F^p H^{2p}(X,\mathbb{C})$.

$cl(Y) \in H^{2p}(X, \underset{\mathbb{Z}}{\mathbb{R}}) \Rightarrow cl(Y) \in F^p H^{2p}(X,\mathbb{C}) \cap H^{2p}(X,\mathbb{Z}) \Rightarrow$
$\Rightarrow cl(Y) \in H^{p,p}(X)$.

Hodge conjecture

If X is a smooth, comp. proj. man. and $Z^p(X,\mathbb{Q})$ is the group of rational codim. p alg. cycles on X then

$$cl : \underbrace{Z^p(X,\mathbb{Q})}_{CH^p(X,\mathbb{Q})} \twoheadrightarrow F^p H^{2p}(X,\mathbb{C}) \cap H^{2p}(X,\mathbb{Q}).$$