Recall: period map: $X \xrightarrow{f} S$, X, S smooth quasi-proj., f proper, surj. of max rank $\rightsquigarrow \varphi: S \to D/\Gamma$.
$s \mapsto H^k(X_s, \mathbb{C})$, $X_s = f^{-1}(s)$

1) φ is locally liftable;
2) φ is holo. $\left(\frac{\partial F^p}{\partial \bar{s}_j} \subseteq F^p\right)$;
3) φ satisfies Griffiths horizontal transversality $\left(\frac{\partial F^p}{\partial s_j} \subseteq F^{p-1}\right)$.

$p_0 \in S$, $0 \to \Theta(X_{p_0}) \to \Theta(X)_{|X_{p_0}} \to N \to 0 \Rightarrow$ (Θ: holo. tangent vector fields)

$$\Rightarrow H^0(X_{p_0}, N) \xrightarrow{\partial} H^1(X_{p_0}, \Theta(X_{p_0})) \to \underbrace{H^1(X_{p_0}, \Theta(X)_{|X_{p_0}})}_{=0 \text{ ideally}}$$

$f_*: T_{p_0}S \to H^0(X_{p_0}, N)$ well def., $\delta: T_{p_0}S \to H^1(X_{p_0}, \Theta(X_{p_0}))$ it exists.

$v \in T_{p_0}S \dashrightarrow \delta(v) \in H^1(X_{p_0}, \Theta(X_{p_0}))$

$$H^{p,q}(X_{p_0}) \cong H^q(X_{p_0}, \Omega^p)$$

$$\underbrace{H^1(X_{p_0}, \Theta(X_{p_0}))}_{T_pS} \times \underbrace{H^q(X_{p_0}, \Omega^p)}_{\underbrace{H^{p,q}(X_{p_0})}_{F^p}} \xrightarrow{\text{contract}} H^{q+1}(X_{p_0}, \Omega^{p-1}) \cong \underbrace{H^{p-1,q+1}(X_{p_0})}_{F^{p-1}}$$

X smooth proj. over $\mathbb{C}$
H Hodge structure of weight $2N-1$ (ex.: $H = H^{2N-1}(X, \mathbb{C})$)
$H_{\mathbb{R}}/H_{\mathbb{Z}} = J(H) \cong (S^1)^l$
$H_{\mathbb{C}} = F^N H_{\mathbb{C}} \oplus \overline{F^{(2N-1)-N+1} H_{\mathbb{C}}} = F^N H_{\mathbb{C}} \oplus \overline{F^N H_{\mathbb{C}}}$.
$H_{\mathbb{R}}/H_{\mathbb{Z}} \cong \dfrac{H_{\mathbb{C}}}{F^N H_{\mathbb{C}} + H_{\mathbb{Z}}}$: $H_{\mathbb{R}} = \{\alpha + \bar{\alpha} \mid \alpha \in F^N H_{\mathbb{C}}\}$.

$C_{|H^{p,q}} = i^{p-q}$, $C: H_{\mathbb{C}} \to H_{\mathbb{C}}$; $C: H_{\mathbb{R}} \to H_{\mathbb{R}}$: $\alpha = \sum \alpha^{p,q} \in H_{\mathbb{R}} \Rightarrow \alpha^{q,p} = \overline{\alpha^{p,q}} \Rightarrow$
$\Rightarrow \overline{C\alpha} = \overline{\sum i^{p-q} \alpha^{p,q}} = \sum i^{q-p} \alpha^{q,p} = C\alpha$.
$C^2 \alpha^{p,q} = (-1)^{p-q} \alpha^{p,q} = -\alpha^{p,q}$ ($p+q = 2N-1 \Rightarrow p-q$ odd) $\rightsquigarrow$ complex structure on $H_{\mathbb{R}}$.

Lucky case: $H^{2p-1}(X, \mathbb{C}) = H^{p,p-1}(X) \oplus H^{p-1,p}(X)$
(ex.: X curve, $p=1$).
Ex.: cubic 3-folds $\rightsquigarrow H^3(X, \mathbb{C}) = H^{2,1}(X) \oplus H^{1,2}(X)$.
Ex.: C curve, $\tilde{C} \xrightarrow{\pi} C$ 2:1 etale double cover of C.
$\pi^*: H^1(C) \to H^1(\tilde{C})$, $\pi^*: J(H^1(C)) \to J(H^1(\tilde{C})) \rightsquigarrow$
$\rightsquigarrow P(C) = \dfrac{J(H^1(\tilde{C}))}{\pi^*(J(H^1(C)))}$; $\dim P(C) = ?$

$\chi(\tilde{C}) = 2\chi(C) \Rightarrow g(\tilde{C}) = 2g(C) - 1$.
$\dim J(H^1(\tilde{C})) = g(\tilde{C}) = 2g(C) - 1$,
$\dim J(H^1(C)) = g(C) \Rightarrow$
$\Rightarrow \dim P(C) = g(C) - 1$.

X smooth 3-fold $\Rightarrow H^{3,0}(X) = 0$, $H^{2,1}(X)$ has dim. $5 \Rightarrow$
$\Rightarrow J(H^3(X))$ has dim. 5. $\cong P(C)$? $5 = g(C) - 1 \Rightarrow g(C) = 6$?
If C is of degree d in $\mathbb{P}^2$, $g(C) = \frac{(d-1)(d-2)}{2} \rightsquigarrow d = 5$.
$X \subseteq \mathbb{P}^4$ smooth cubic 3-fold $\Rightarrow$ contains a line $l \leftrightarrow l_1, l_2, l_3 : \mathbb{C}^5 \to \mathbb{C}$.
$X \ni p \dashrightarrow [l_1(p) : l_2(p) : l_3(p)] \subseteq \mathbb{P}^2$.
$\mathbb{P}^2 \supseteq C$ = locus where preimages of projection from l consists of union of 2 lines.